weird weaving - Principles of Construction

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1 Introduction

The main principles of this piece are the following:

1. All permutations of a set of intervals. 3 intervals make $3! = 6$ permutations.
2. All permutations of a set of note lengths. 4 notes with different lengths make $4! = 24$ permutations.
3. All combinations of elements from a set of articulations.
4. Principle of minimum changes for the next permutation or combination.

All note length permutations combined with all interval permutations gives $24 \cdot 6 = 144$ combinations of 4 notes each, making $4 \cdot 144 = 576$ notes. These notes are grouped into bars of 3 notes each, that is $576/3 = 192$ bars.

The sequence of the 144 combinations is determined by a Hilbert curve in accordance to the fourth principle. This leads to a weird path in the plane (see green line in Figure 1 below). Hence the title "weird weaving".

2 Plain Notes

First the construction of a sequence of plain notes is described. A plain note is defined by a pitch and a note length.

The set of note lengths is the following:

$$1/8, 2/8 = 1/4, 3/8, 5/8$$

Note the Fibonacci numbers $1, 2, 3, 5$.

There are $4! = 24$ different permutations of these four note lengths. They are ordered as follows (with l as an order index):

l	note lengths permutation
0	1/8, 1/4, 3/8, 5/8
1	1/8, 1/4, 5/8, 3/8
2	1/8, 5/8, 1/4, 3/8
3	5/8, 1/8, 1/4, 3/8
4	5/8, 1/8, 3/8, 1/4
5	1/8, 5/8, 3/8, 1/4
6	1/8, 3/8, 5/8, 1/4
7	1/8, 3/8, 1/4, 5/8
8	3/8, 1/8, 1/4, 5/8
9	3/8, 1/8, 5/8, 1/4
10	3/8, 5/8, 1/8, 1/4
11	5/8, 3/8, 1/8, 1/4
12	5/8, 3/8, 1/4, 1/8
13	3/8, 5/8, 1/4, 1/8
14	3/8, 1/4, 5/8, 1/8
15	3/8, 1/4, 1/8, 5/8
16	1/4, 3/8, 1/8, 5/8
17	1/4, 3/8, 5/8, 1/8
18	1/4, 5/8, 3/8, 1/8
19	5/8, 1/4, 3/8, 1/8
20	5/8, 1/4, 1/8, 3/8
21	1/4, 5/8, 1/8, 3/8
22	1/4, 1/8, 5/8, 3/8
23	1/4, 1/8, 3/8, 5/8

The ordering is such that two successive permutations differ only in a swap of two neighboring elements (in accordance with the fourth principle). This is achieved by the Steinhaus-Johnson-Trotter (SJT) algorithm (for details see https://en.wikipedia.org/wiki/Steinhaus-Johnson-Trotter_algorithm).

The set of intervals is the following (in units of half-tones): $2, -3, 2.5$

There are $3! = 6$ different permutations. Again ordered by the SJT algorithm gives (with i as an order index):

i	intervals permutation
0	2, -3, 2.5
1	2, 2.5, -3
2	2.5, 2, -3
3	2.5, -3, 2
4	-3, 2.5, 2
5	-3, 2, 2.5

For each note lengths permutation l and each intervals permutation i a group of four notes is generated. That is, a sequence of $576 = 4! \cdot 3! \cdot 4$ plain notes is generated in $4! \cdot 3! = 144$ groups. For each group the initial pitch is given by

$$55 + \text{floor}(l \cdot i/4)/2$$

where floor is the function which returns the largest integer $\leq$ the function argument.

The 144 groups can be imagined as tiles of a rectangle where the indices l and i define the position of a tile in this rectangle. In accordance with the fourth principle the next group should be either a different note lengths permutation or an intervals permutation. The chosen permutation should be either before or after the current permutation in the lists above. That is, the tiles of the rectangle should be scanned in such a way that the next tile is either the tile above, below, left or right. There are many ways to do such a scan which visits all tiles but never visits a tile more than once. A generalization of the Hilbert curve has been chosen.

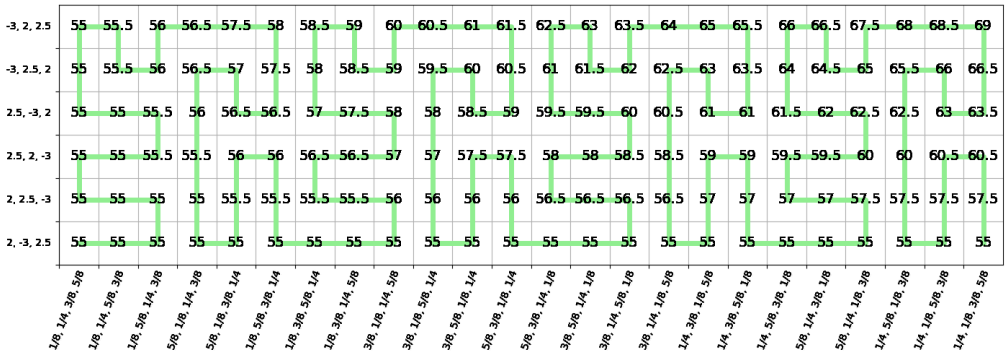

Figure 1: This diagram shows the 144 tiles with initial pitch in the center. The x axis shows the note lengths permutations. The y axis shows the interval permutations. The green path is the generalized Hilbert curve of the scan. It starts in the lower left corner and ends in the lower right corner.

3 Articulations

The sequence of 576 plain notes is cut into phrases of three notes. There are $576/3 = 192$ phrases. For each phrase a pause of $\geq 1/8$ is append such that the total length of the phrase (i.e. the three plain notes plus the pause) is an integer multiple of a quarter.

The following articulations are used:

1. note: p, f, f & stacc., p & stacc.
2. note: p, f, f & stacc., p & stacc., p & tongue pizz., f & tongue pizz., f & tremolo, p & tremolo
3. note: p & stacc., f & stacc., f, p, p & tremolo, f & tremolo

where p and f stand for relativ dynamics. At the beginning of the piece p is translated to *pp* and f to *mf*. At the end they are translated to *mp* and *ff*, respectively. The articulation "stacc." stands for "staccato" and is either a staccato on the note if the note length is $1/8$ or the original length is reduced to $1/8$ replacing the rest by a pause.

The total number of combinations of the different articulations of the three notes is $4 \cdot 8 \cdot 6 = 192$. Each articulation combination appears only once. For two successive phrases the articulation differs only in one note, again in accordance with the fourth principle. To achieve this a non-binary Gray code (see https://en.wikipedia.org/wiki/Gray_code#n-ary_Gray_code) has been used.

There is always a crescendo or decrescendo if a long note is followed by a note of different dynamics.

A phrase is accompanied by singing in the case of absence of staccati, tremoli and tongue pizzicati.